Analytical beam profile solution for rastered Gaussian beam

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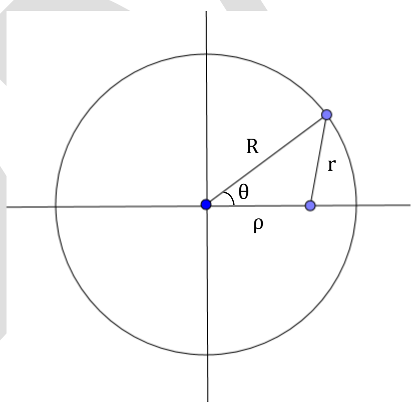
The 2D Gaussian function is given by Equations [1] and [2].

$$f(x, y) = \frac{A}{2\pi\sigma^2} e^{\left[\frac{-x^2 - y^2}{2\sigma^2}\right]} \qquad [1]$$

$$f(r) = \frac{A}{2\pi\sigma^2} e^{\left[\frac{-r^2}{2\sigma^2}\right]} \qquad [2]$$

A: number of protons, x: x-coordinate (cm), y: y-coordinate (cm), σ: beam sigma (cm), r: radial coordinate (cm)



r: local radial coordinate of 2D Gaussian (cm)
ρ: global radial coordinate (cm)
R: sweep radius (cm)

The 2D Gaussian function [2] is then integrated with respect to θ, and the average obtained by dividing by 2π,

$$f(\rho, r) = \frac{A}{4\pi^2}\int_0^{2\pi} \frac{1}{\sigma^2} e^{\left[\frac{-r(\theta)^2}{2\sigma^2}\right]} d\theta \qquad [3]$$

Using the cosine rule, Equation 4 expresses r as a function of θ and ρ,

$$r(\theta, \rho) = [\rho^2 + R^2 - 2\rho R\cos\theta]^{\frac{1}{2}} \qquad [4]$$

Substituting [4] in [3],

$$f(\rho, R) = \frac{A}{4\pi^2}\int_0^{2\pi} \frac{1}{\sigma^2} e^{\left[\frac{-\rho^2 - R^2 + 2\rho R\cos\theta}{2\sigma^2}\right]} d\theta \qquad [5]$$

$$f(\rho, R) = \frac{A}{4\pi^2\sigma^2} e^{\left[\frac{-\rho^2 - R^2}{2\sigma^2}\right]} \int_0^{2\pi} e^{\left[\frac{2\rho R\cos\theta}{2\sigma^2}\right]} d\theta \qquad [6]$$

$$\int_0^{2\pi} e^{\left[\frac{2\rho R\cos\theta}{2\sigma^2}\right]} d\theta = 2\pi I_0\left(\frac{\rho R}{\sigma^2}\right) \qquad [7]$$

where I_0 is the modified Bessel function of the first kind

$$f(\rho, R) = \frac{A}{2\pi\sigma^2} e^{\left[\frac{-\rho^2 - R^2}{2\sigma^2}\right]} I_0\left(\frac{\rho R}{\sigma^2}\right) \qquad [8]$$

f(ρ,R): proton intensity (p/cm^2)